Computer Simulation of Particle Size Classification in Air Separators

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ABSTRACT: *Cement powder size classification efficiency significantly affects quality of final product and extent of energy consumption in clinker grinding circuits. Static and dynamic or high efficiency air separators are being used widely in closed circuit with multi-compartment tube ball mills, High Pressure Grinding Rolls (HPGR) and more recently Vertical Roller Mills (VRM) units in cement plants to classify comminuted clinker particles at finish grinding stage. Therefore, simulation of air separators is of critical importance in order to provide tools that can assist cement plants engineers in their routine clinker grinding circuit optimization efforts. In this paper, Air Separator Simulator (ASSIM), a newly developed simulator implemented in VB™ which provides a user-friendly process analysis and optimization environment will be introduced. First, a review of mathematical modeling of cyclone separators is presented. Then, the details of ASSIM and the results of its testing using industrial data from J. K. White Cement Works plant will be discussed. The simulator is mainly based on the Whiten function to model air separators and predicts fine and coarse output streams particle size distributions and flow rates. ASSIM performance was verified and validated by comparing its outputs with measured data collected around an operating air separator. Preliminary software tests indicate the accuracy and precision of the developed code in predicting various properties of output streams as sum of least squares between predicted results and actual data is less than 0.01.*

KEY WORDS: *Air separators, Cement size classification, Classification simulation, Air separators modeling, ASSIM.*

INTRODUCTION

Grinding process is the most energy-consuming operation in cement production. About 30 % of the total electrical energy required to produce one tone of cement product is used in raw material preparation and 40 % is used in clinker finish grinding [1]. There is no doubt that air separators installed and operated in dry clinker grinding

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circuits have strong impact on circuit capacity, energy consumption and quality of final product. Applying effective strategies to optimize operating parameters of air separators such as percent of feed solids in the air, rotor speed and percent of fan opening is of critical importance if a more efficient cement grinding operation is sought. In the past several decades, numerous research studies have been carried out to improve grinding and classification performances by introducing accurate and well-proven mathematical models. *Zhang* published the results of first attempts to use a new empirical function to model efficiency curves of air separators used in cement industry [5]. In recent years, *Benzer* and his colleagues have done a number of research projects to simulate cement grinding circuits. They used Whiten model to simulate air separators which resulted in accurate predictions of fine and coarse streams particle size distributions [6-8]. Although, most of these studies are done in the field of static simulation by introducing different models, studies on dynamic simulation essential for describing processes in some devices are in progress [9].

Tremendous progress in computer hardware and software technologies have made it possible that powerful commercial software such as, JKSimMet™, MODSIM™, USIM PAC™ to be developed as off-line mineral processing optimization tools. Authors also have developed a number of dedicated software tools including BMCS, NGOTC, BFDS and COMSIM [2-4] mainly for ball milling circuit simulation. Direct applications of these softwares to optimize cement clinker grinding circuits is faced with some limitations due to the special models and simulation structures required for describing multi-compartment tube ball mills and air separator devices used in cement production plants. Presently, JKSimMet™ incorporates air separator models which are required for simulation of cement clinker grinding circuits.

In this paper, the results of a literature review on mathematical modeling of air separators and development of a new software, ASSIM, is reported. Three best known modeling techniques are first introduced. Then, the Whiten model used in developing ASSIM is discussed in details and compared with other similar models. ASSIM relies on capabilities of Visual Basic programming language to provide an easy-to-use process analysis and optimization environment.

AIR SEPARATORS MODELS

Various modeling methods have been developed to describe air separators' performance and simulate their operations. In the following sections three main types of mathematical models, i.e., those which are based on Discrete Element Methods (DEM), Computational Fluid Dynamics (CFD) and empirical relationships are explained.

Discrete Element Method (DEM)

This method considers factors affecting discrete particles in any processing equipment. In any air classification process, there are three main acting forces which cause separation of particles into coarse and fine streams. They are called gravity, centrifugal and air drag forces. It is the balance of these three forces which determines classification cut size (d_{50}). Centrifugal (F_c) and air drag (F_d) forces for a spherical shape are as follow [10]:

$$F_c = \frac{4}{3} * \pi * r_p^3 * \rho_p * \frac{\upsilon^2}{r} \tag{1}$$

$$F_d = c_x * \rho * \pi * r_p^2 * \frac{\upsilon_a^2}{2} \tag{2}$$

DEM technique is increasingly used for describing breakage operation in tumbling mills. This technique has been recently used for simulating and visualizing the motion of particles inside air separators. However, relations described above are time-consuming and normally applied to discuss parameters affecting efficient separation process and are not used to predict products' size distributions.

Computational Fluid Dynamics (CFD)

Karunakumari described computational fluid dynamics (CFD) as "a method that deals with the numerical solutions of the governing equations and constitutive laws of fluid flows and enables fairly accurate calculation of a single-phase flow field in equipment of arbitrary geometric complexity" [11].

The application of CFD technique to simulate cyclones and other types of gas-solid separators is also well established in the other literatures [12-14]. *Karunakumari* and his colleagues were successful in using this approach and predicted results were close to the observed data [11].

Use of CFD method is increasing as this approach of modeling is able to provide useful predictions [15], but it is so much time consuming as Nageswararao in his recent paper mentioned “Such solutions are computationally intensive; current JKMRC work on the CFD modeling of a hydrocyclone operating under normal industrial conditions using parallel processing in a super computer can consume two weeks of CPU time for one steady-state simulation” [16]. Therefore, at the time being, accurate predictions of empirical models that provide simple equations are preferred by developers of simulators.

Empirical Models

Empirical relationships have been used in most steady-state simulators to model classification in hydrocyclone devices. However, empirical models require sampling from actual circuit for calibration and validation before being used for simulation. Authors implemented an algorithm based on this approach of modeling due to its sufficient accuracy, high execution speed and lower use of computer resources during simulation runs. The usual approach to describe performance of air separators uses the concept of efficiency curves, known as selectivity curves (Fig. 1). Efficiency curves can be plotted for the percent of mass in a specific particle size class in feed stream which is recovered into either coarse or fine streams. The two types of curves are complementary. A sharper curve represents a more efficient separation.

Equations (3) and (4) are used to plot efficiency curves based on mass-balanced particle size distribution data and operating conditions. This requires sampling from feed, coarse and fine streams around the air separator [17].

$$E_{oa} = 100\left(\frac{O}{F}\frac{o_i}{f_i}\right) \tag{3}$$

$$E_{ua} = 100\left(\frac{U}{F}\frac{u_i}{f_i}\right) \tag{4}$$

It can be seen in Fig. 1 that efficiency curve does not cross the origin at zero-sized particles. The offset on the vertical axis is due to the “bypass or short circuiting” that may be observed in size classification devices since small particles do not respond to classification forces and therefore enter coarser stream as misplaced materials.

Empirical modeling of air separator devices is closely related to the research that has been done for modeling of hydrocyclones. In order to simulate air separators for design or optimization objectives, some researchers have applied the fitting models that have already been used to describe efficiency curves of size separation units. Among the mathematical functions proposed for fitting observed efficiency curve data, those of Plitt/Reid based on Rosin-Rammler distribution and Whiten based on hyperbolic function have been widely accepted and used by mineral processors [17]. Plitt/Reid function to calculate fractional recovery to underflow (coarse) stream is as follows [17]:

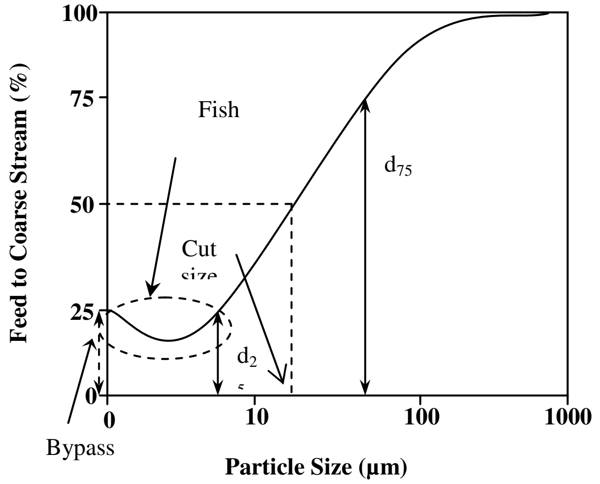


Fig. 1: Typical shape of efficiency curve for recovery to coarse strea.

$$E_{uc} = 1 - \exp\left[-0.693\left(\frac{d}{d_{50c}}\right)^m\right] \tag{5}$$

By converting Plitt model to a linear relation, model parameters are easily determined [18]. Whiten used an exponential expression to develop a function for fitting efficiency curves describing solids recovery to overflow stream. Whiten function is as follows [17]:

$$E_{oa} = C\left(\frac{\exp(\alpha)-1}{\exp(\alpha X)+\exp(\alpha)-2}\right) \tag{6}$$

A controversial subject in performance analysis of size classification devices is observation of efficiency curves with a portion similar to fish hooks at fine end of some efficiency curves. Several researchers have contributed to the subject of fish-hook modeling [19-25]. It is noted that when finer cement product is needed, it is very likely to observe the fish-hook effect.

Majumder and *Shah* experimentally verified and asserted that fish hook is a reproducible phenomenon which can be observed in the selectivity curve of any centrifugal separator [23-25]. Therefore, modeling fish hook is necessary in accurate simulation of centrifugal classifiers such as hydrocyclones and air separators. Three general models which can be used to fit efficiency curves of hydrocyclones or air separators will be discussed in below.

Finch assumed that the mass of particles reporting to coarse stream independent of classification forces due to entrainment in water will increase with decreasing particles size [19]. For simplicity, he used a linear model to describe this phenomenon and added a new term to the right side of the Eq. (5) to obtain a model which includes fish-hook effect:

$$E_{uc} = 1 - \exp\left[-0.693\left(\frac{d}{d_{50c}}\right)^m\right] + R_f\left(\frac{d_o - d}{d_o}\right) \qquad (7)$$

Based on the argument that contributions of short circuiting and true classification to solids recovery to underflow are probabilistic, *Del Villar* and *Finch* modified Eq. (7) and proposed a final fish-hook cyclone model [20]:

$$E_{uc} = \left\{\left[1 - R_f\left(\frac{d_0 - d}{d_0}\right)\right]\right\}\left\{\left[1 - \exp\left(-0.693\frac{d}{d_{50_c}}\right)^m\right]\right\} + \qquad (8)$$
$$R_f\left(\frac{d_0 - d}{d_0}\right)$$

Zhang used Eq. (9) to describe the efficiency curve of air classifiers [5]:

$$E(X) = 100 \times C \frac{e^{-\alpha X}}{AX^2 - BX + 1.0} \qquad (9)$$

Equation (9) can also model the fish-hook phenomenon. Whiten function is another proposed model that can fit efficiency curve data with fish hook behavior at fine sizes [17]:

$$E_{oa} = C\left[\frac{(1+\beta.\beta^*.X)(\exp(\alpha)-1)}{\exp(\alpha.\beta^*.X)+\exp(\alpha)-2}\right] \qquad (10)$$

Equation (10) can be used to fit any actual efficiency curve. For simple form of efficiency curve, having no fish hook phenomenon, β equals with zero when β^* equals with 1. Values of parameters α, β, β^*, C and d_{50c} are determined by back calculation procedures known as non-linear regression being discussed in mathematics book.

In recent past years, validity and accuracy of Whiten model predictions have been further tested in various case studies carried out by *Benzer* and his colleagues [6-8, 26]. They successfully applied Whiten model to simulate air classification in static and dynamic air separators installed in a number of cement plants. A property of Whiten function is that its parameters are independent of operating conditions, so this feature makes this function more helpful in optimizing operating conditions.

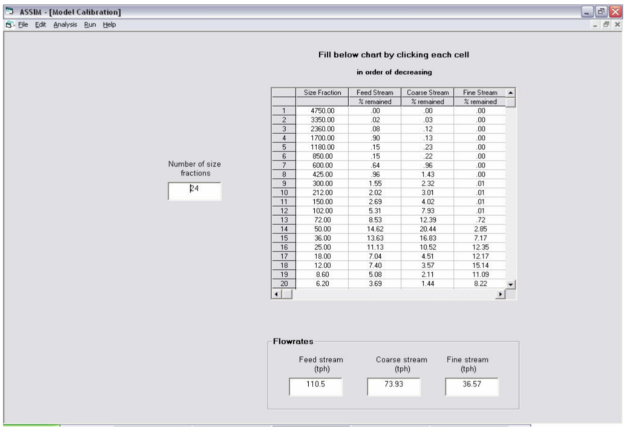
Authors have developed and tested ASSIM as a tool for air separator model calibration and simulation based on the Whiten function. The program can be used to analyze the performance of an installed air separator by computing the optimal values of efficiency curve parameters (such as cut size, separation sharpness and by-pass) or to simulate the product size distribution and flow rates of fine and coarse streams based on the calibrated efficiency curve and feed data. By linking ASSIM with ball mill or HPGR models within a simulator structure, whole circuit simulations would be possible.

ASSIM DEVELOPMENT

ASSIM is implemented in Visual Basic Programming language and uses unique features of this language to create a user-friendly environment in Microsoft™ Windows™. There are five main menus at the top of the window similar to commonly used software that provide usual functionalities such as saving, opening and editing data files, analyzing entered data and displaying predicted results by different tables and graphs.

ASSIM Overview

ASSIM predicts characteristics of fine and coarse streams of air separators using a previously calibrated Whiten model as discussed in section of Empirical model. The values of parameters which define Whiten function are unique for any air separator and at least one measured data set (which is obtained by size analysis of samples from three streams around the air separator and recorded operating condition during the sampling campaign) is required to estimate them. As discussed before, nonlinear regression is used to estimate whiten parameters using



Fill below chart by clicking each cell

in order of decreasing

Number of size fractions: 24

	Size Fraction	Feed Stream % remained	Coarse Stream % remained	Fine Stream % remained
1	4750.00	.00	.00	.00
2	3350.00	.02	.03	.00
3	2360.00	.08	.12	.00
4	1700.00	.90	.13	.00
5	1180.00	.15	.23	.00
6	850.00	.15	.22	.00
7	600.00	.64	.96	.00
8	425.00	.96	1.43	.00
9	300.00	1.55	2.32	.01
10	212.00	2.02	3.01	.01
11	150.00	2.69	4.02	.01
12	102.00	5.31	7.93	.01
13	72.00	8.53	12.39	.72
14	50.00	14.62	20.44	2.85
15	36.00	13.63	16.83	7.17
16	25.00	11.13	10.52	12.35
17	18.00	7.04	4.51	12.17
18	12.00	7.40	3.57	15.14
19	8.60	5.08	2.11	11.09
20	6.20	3.69	1.44	8.22

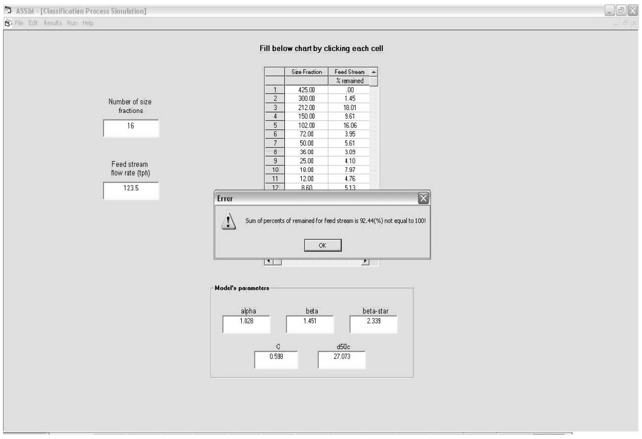
Flowrates

Feed stream (tph): 110.5

Coarse stream (tph): 73.93

Fine stream (tph): 36.57

Fig. 2: The dialog box for model calibration data entry.



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Number of size fractions: 16

Feed stream flow rate (tph): 123.5

	Size Fraction	Feed Stream % remained
1	425.00	.00
2	300.00	1.45
3	212.00	18.01
4	150.00	9.61
5	102.00	16.06
6	72.00	3.95
7	50.00	5.61
8	36.00	3.09
9	25.00	4.10
10	18.00	7.97
11	12.00	4.76
12	8.60	5.13

Error

Sum of percents of remained for feed stream is 92.44(%) not equal to 100!

Model's parameters

alpha: 1.028

beta: 1.451

beta-star: 2.339

C: 0.598

d50c: 27.073

Fig. 3: An error message displayed by ASSIM.

sampled data. In this method starts from an estimate for each parameter and continues by back calculations in order to find the best value for the parameters which has the least squares between measured and calculated data. ASSIM uses an algorithm for estimating Whiten function's parameters in order to calibrated each air separator.

The main functions of ASSIM are "Model Calibration" and "Simulation". Simulation trials can be done directly if the parameters of Whiten model are already known; otherwise, model calibration is necessary to build a customized air separator model for simulation. In order to run model calibration procedure, the number of size fractions and flow rates of the three streams around the air separator should be entered in text boxes available for this purpose. Size fractions and size distributions of the three streams in percent of mass remained in each fraction should be entered in a table available in the window (Fig. 2).

By clicking "View" menu, user is able to view the calibrated values of Whiten function's parameters; the user can view simulator's analysis of data entered for model calibration by different graphs and tables of size distributions and efficiency curves. By clicking "Run| Classification simulation" menu, user can enter simulation step where feed stream properties including the number of size fractions, mass flow and particle size distribution should be entered. As mentioned before, parameters of Whiten function is independent of operating parameters. Therefore, the user can change the operating parameters in order to simulate the classification process and see the resulting characteristics of fine and coarse streams.

Simulation results in forms of graphs and tables of size distributions and recovery of coarse and fine streams can be seen by clicking "Results" menu. As ASSIM checks validity of the input data, any errors would be informed to the user and should be removed prior to start any other process. Fig. 3 shows an error message shown when sum of size distributions of feed stream in worksheet of simulation data entry do not equal with 100 percent. Though ASSIM checks validity of data entered by different tools, it is essential to enter mass balanced data to have the most accurate and reliable results. Data should be entered in order of ascending in both worksheet of data entry

Case Study and Software Validation

During development of ASSIM, different sets of data collected from a number of air separators operating in cement plants were used to verify software computations, non-linear algorithm and its results. Here, an industrial case study is presented which constitute the developing database used for ASSIM validation. The performance of a real air separator was simulated. Plant data were supplied by J. K. White Cement Works located at Gotan in Rajasthan State of India.

Fig. 4 shows the simplified flowsheet of J. K. White Cement Works clinker grinding circuit. The fresh feed (clinker mixed with high-grade gypsum) is distributed between the two parallel cement tube ball mills which are operated in a closed circuit with a single high-efficiency dynamic air separator. The two datasets were obtained by particle size analysis of the samples collected from the streams around the air separator. Particle size determination at sub-sieve range (less than 40 µm) was done by laser beam diffraction method. Table 1 shows operating conditions at the time of sampling campaigns.

The samples collected under two operating conditions which are completely different in terms of feed particle size distribution and flow rate. Therefore, the first data set was used to fit the actual efficiency curve by finding the optimal values of Whiten model parameters. The obtained set of values for various parameters is used to indicate air separator performance for classification process analysis purposes and to customize and calibrate Whiten model for classification process simulation.

The measured data, i.e. particles size distributions and solids flow rates of all streams around the air separator, were first adjusted using NORBAL3 material balance reconciliation software [27]. Solids flow rates for fine and coarse streams were calculated based on the measured mass flow rates of the fresh feed and average circulating load to the tube ball mills.

The values of Whiten model parameters found by running model calibration function are automatically exported to the classification simulation part of the software. The efficiency curves of the air separator computed based on the first measured dataset and Whiten model are compared in Fig. 5. The fish-hook phenomenon is easily noticeable in these curves, which occurs at an onset particle size approximately equal to 9 μm. The quality of fitting the fish-hook part of the measured efficiency data by the implemented Whiten model is quite significant. Predications of ASSIM were then checked and validated using the second data set. Therefore, flow rate and size distribution of feed stream are entered to run air classification simulation step. Table 2 shows observed and predicted particle size distributions and flow rates for fine and coarse streams. The predicted solids flow rates of fine and coarse output streams were equal to 33.8 and 89.70 t/h, respectively. These values are very close to the observed solids flow rates (33.54 and 89.96 t/h for fine and coarse streams, respectively). Particle size distribution curves of both output streams have been also compared with the measured data in Fig. 6.

The close agreement of predicted particle size distributions with the measured data indicates the accuracy of the implemented model and reliability of the developed source code.

CONCLUSIONS

Comparisons of ASSIM predictions with real measured data validated the results generated by simulator program.

Table 1: Operating conditions during two sampling program from clinker grinding circuit at J.K. White Cement Works.

	1st dataset	2nd dataset
Date of Sampling	19/10/2006	31/10/2006
Mills feed rate (t/h)	22, 12	24, 14
Total feed rate (t/h)	34	38
Outlet draught (mbar)	5.2, 5.1	7.3, 4.2
Cement temperature at mill outlets (°C)	109, 108	102, 122
Mills motor power draw (kW)	809, 433	784, 486
Separator fan speed (rpm)	910	905
Separator Rotor speed (rpm)	1060	1080

Table 2: Observed and predicted particle size distributions of output streams around air separator of J. K. White Cement Works.

Particle	Size Distribution			
(μm)	Observed	ASSIM	Observed	ASSIM
	Fine stream	Fine	Coarse	Coarse
425	100.00	100.00	100.00	100.00
300	100.00	100.00	98.01	98.00
212	100.00	100.00	73.28	73.21
150	100.00	100.00	60.09	59.98
102	99.98	99.98	38.05	37.87
72	99.88	99.88	32.66	32.47
50	98.87	98.85	25.34	25.13
36	96.93	96.88	21.82	21.62
25	91.33	91.29	18.28	18.08
18	75.04	74.91	13.41	13.29
12	63.20	63.03	11.29	11.21
8.6	49.47	49.47	9.37	9.25
6.2	40.00	40.02	7.97	7.87
4.4	30.40	30.43	6.43	6.35
3	25.22	25.29	5.52	5.44
1.8	17.39	17.38	3.90	3.86

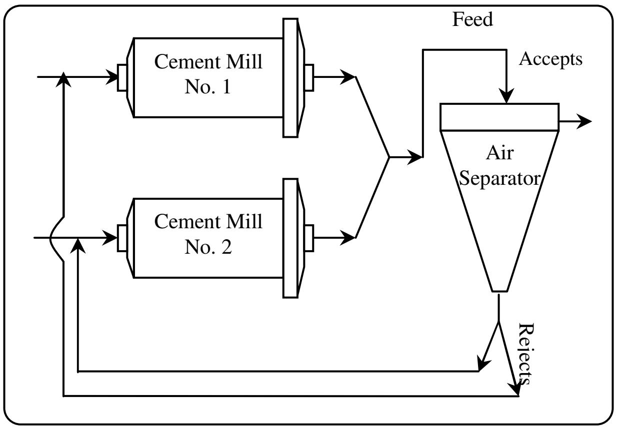

Fig. 4: J. K. White Cement Works grinding circuit flowsheet.

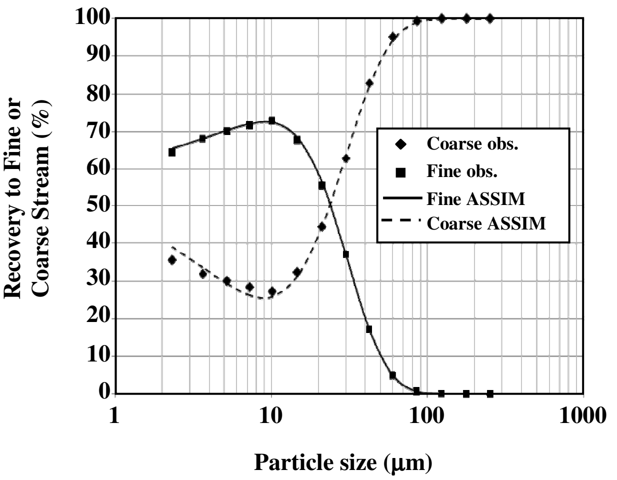

Fig. 5: Measured and fitted efficiency curves plots for air separator of J.K. White Cement Works.

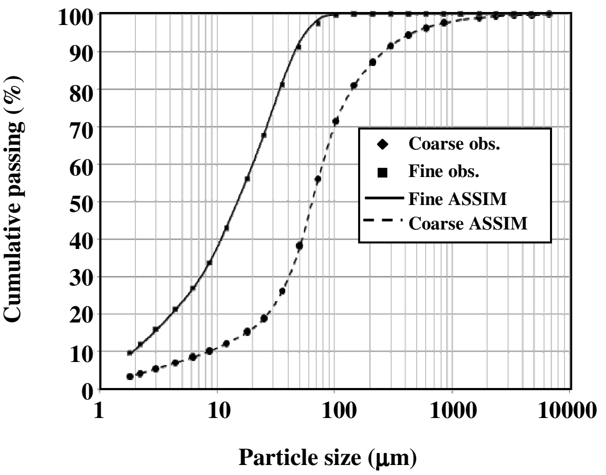

Fig. 6: Comparison of predicted and measured particle size distribution curves of air separator output streams at J.K. White Cement Works.

ASSIM uses Whiten model, previously implemented in JKSimMet^{TM} software developed at JKMRC, to simulate the classification process. Once the model is calibrated using air separator efficiency curve data under specific steady-state operating conditions, its performance for new operating conditions can be simulated. ASSIM can be used by process engineers as a standalone computer program to analyze and simulate classification performance. ASSIM can be used for the purpose of operating parameters calibration by comparing outputs' results under different operating conditions. This software is in progress and the future work includes linking the air separator model implemented in ASSIM with the multi-compartment tube ball mill model in order to simulate cement closed grinding circuits.

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Nomenclatures

ρ_p	Density of the particle
r_p	Radius of the particle
υ	peripheral velocity of the rotor
υ_a	Air velocity
P	Air density
c_x	Drag coefficient
E_{oa}	Percent of feed material entered into fine stream (actual curve)
E_{ua}	Percent of feed material entered into coarse stream (actual curve)
O	Fine flow rate (tph)
U	Coarse flow rate (tph)
F	Feed flow rate (tph)
o_i	Percent of a specified particle size "i" in fine stream
u_i	Percent of a specified particle size "i" in coarse stream
f_i	Percent of a specified particle size "i" in feed stream
E_{uc}	Fraction of feed reporting to underflow (corrected curve)
d	Particle size
d_{50c}	Corrected cut size
m	Sharpness of separation
C	Fraction of material that is usually classified (1-Rf)
a	Separation sharpness parameter
X	Normalized particle size (d/d50c)
d_o	The size below which entrainment occurs
R_f	Percent of bypass
B	Fish-hook parameter
β^*	Parameter that preserve definition of d50c parameter (when Eoa=C/2, d=d50c)

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