

A New Approach for Constructing Pore Network Model of Two Phase Flow in Porous Media

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ABSTRACT: *Development of pore network models for real porous media requires a detailed understanding of physical processes occurring on the microscopic scale and a complete description of porous media morphology. In this study, the microstructure of porous media has been represented by three dimensional networks of interconnected pores and throats which are designed by an object oriented approach. Afterwards, the connectivity of the system has been optimized by an optimization algorithm. To validate the methodology, a network of a carbonate sample is constructed. In this model, the geometrical characteristics of the pores and throats, such as their shapes, effective radii and lengths, are selected from the image analysis of SEM picture and statistical distribution methods based on the mercury injection test results. Then the constructed network is further tuned according to laboratory measured porosity, absolute permeability and capillary pressure. Having built a flexible and detailed model, its prediction of relative permeability and saturation variation along a core plug are compared with experimental data for both drainage and imbibition phenomena. This comparison shows good matches for almost all experimentally measured data.*

KEY WORDS: *Pore structure, Pore network, Drainage, Imbibition data representation.*

INTRODUCTION

Using network modeling to measure the microscopic properties of heterogeneous porous media is an excellent method in reservoir modeling. It can be used to model complicated phenomena such as mass and energy transfer, hydrology and multiphase flow in porous media. Network modeling was first proposed by *Fatt* in 1956. He used a simple two dimensional 50 by 50 network of regular resistors to generate relative permeability and capillary pressure data for a drainage type displacement [1-3].

Following his original work, this area of research attracted a lot of attention. These efforts mainly have undertaken two distinguished approaches.

The first approach is building the network based on stochastic algorithms through the use of some pore and throat size and shape distribution and its morphological relation and spatial correlation within the system [4, 5]. In the second approach, the geological images of porous media are applied directly to the structure of the network. Therefore, the 3D model is built either by

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combining serial cross sections of 2D images which have been gathered from a specific plug and using measured porosity and special correlation within the system or by using micro tomography magnetic resonance.

The importance of spatial correlations in real porous media was stressed by *Bryant et al.* [6]. They pointed out that the uncorrelated network models are unlikely to be physically representative and that the departure from randomness significantly affects permeability.

Hughes, *Blunt* and *Jackson* [7-9] presented a network model to simulate rate effects using a perturbative approach. The model was used to study the effect of contact angle, capillary number and initial wetting phase saturation on flow patterns, residual non-wetting phase saturations and relative permeability in two or three dimensions.

Kovscek et al. [10] elucidated the pore-scale development of mixed wettability in reservoir media. They proposed the configurations of oil and water relative to pore space and related these to conventional measures of wettability. Their mixed-wettability scenario was used by *Man* and *Jing* [11] when they studied the effects of rock wettability and saturation history on electrical resistivity and capillary characteristics.

Blunt and *valvante* [12-14] studied the effects of long-range pore size correlation on imbibition relative permeability curves using a quasi-static model. They showed that a spatial correlation is necessary to produce qualitatively realistic curves that capture measured hysteresis trends. They also found that changing the contact angle has a significant effect on both residual non-wetting saturation and relative permeability curves. *Blunt* extended this model to incorporate wettability effects based on the work of *Kovscek et al.* [10].

Hiroshi and *Blunt* [15] used multiple-point statistics to generate geologically realistic pore-space representations. They constructed a 3D network based on 2D thin section images as training images to preserve the long-range connectivity of the pore structure. They tested their method using simple sandstone and showed that the long-range connectivity is difficult to reproduce using the traditional two-point stochastic model. The proper multiple-point statistical algorithm is an important factor for this method. Besides the statistical model, the process based models have been developed to include geological processes of rock formation. The pore structure is

constructed based on physical processes.

Vogel and *Roth* [16] first used serial sections through impregnated samples and proposed a connectivity function to measure the spatial connectivity to construct a realistic representation of the three-dimensional pore space. This technique has some limitations because it lacks information regarding the geological processes during the formation of the rock. The essence of their approach is to build pore-scale models that are representative of real sandstone.

Øren and *Bakke* [17] developed a 3D pore-scale model using a new modeling approach. In their model, the results of the main sandstone-forming processes and other petrographical data obtained from 2D thin sections have been used. In this way geological image of porous media has been applied onto a structure of network directly therefore, 3D model built either by combining 2D images Of serial cross sections which have been gathered from specific plug or through the use of measured porosity and special correlation within the system.

Øren and *Pinczewski* [18] used their reconstructed sandstone pore network model to develop a quasi-static model to simulate two-phase primary drainage and water injection for both water-wet and mixed-wet systems. The predicted capillary pressure and relative permeability curves conformed the measured results for water wet systems and they were in a fairly good agreement with the results for mixed-wet systems.

Nguyen et al. [19] developed a dynamic network model for imbibition using a complex treatment of frontal displacement, film swelling, snap-off and snap-off nucleated displacement. They used the model to discover the effect of the displacement rate on imbibition patterns, relative permeability, and residual oil saturations. The details of single phase and multiphase flow have been investigated by *Piri*, *Lopez*, *Valvante* and *Blunt* [20-22]. They found that network modeling is quite capable to model three phase flow under various wettability ranges.

The purpose of this study is to propose a new algorithm for construction of a pore network model to represent a porous medium in a more realistic manner. In this methodology the network connectivity is optimized by using a non-gradient based algorithm and flexible ranges have been applied to main parameters such as

pore and throat size and shape and even fluid-solid contact angle to improve the ability of network to cover the wide range of heterogeneity of the porous media. The pore network model implementation was verified using a carbonate core sample. This pore network provides good matches for all experimentally determined properties. In this paper, first a brief review on pore network model construction algorithms and different mechanisms of fluid flow through porous media are explained. Having gone through this, the experimental information used as the network input and simulation procedure of both drainage and imbibition phenomena are explained in details and finally, the performance of the proposed algorithm is shown through the comparison of model results and available experimental data.

THE NETWORK CONSTRUCTION

For prediction of microscopic properties of porous media by network modeling, a realistic network needs to be constructed. This network must be a representative of the geometry and topology of the pore space. Although the importance of construction of a realistic pore network is known, most pore networks are still based on a regular cubic lattice, typically a cube with a coordination number (number of connected throats to a single pore) of six. The network can also be irregular by using combinations of different coordination numbers and different lengths for throats [23, 24]. With arbitrary distributions of these parameters, the generated network may become more flexible in terms of capturing the complexity of the real porous media. However, in all of these cases the network is still based on a regular topology, whereas a natural porous medium has a more complicated structure.

One of the major advancements in this field of science is finding the topological equivalent of the porous medium. In this method, the three-dimensional network of pores and throats is obtained by a pixilated image of the pore structure at some appropriate resolution. Acknowledging some advantages of this approach, it cannot model the long range connectivity of pore space imposed by geological process, and also from modeling point of view, it is considerably time consuming and expensive [25, 26].

In this study a new methodology is proposed for modeling the complicated nature of porous media. In this

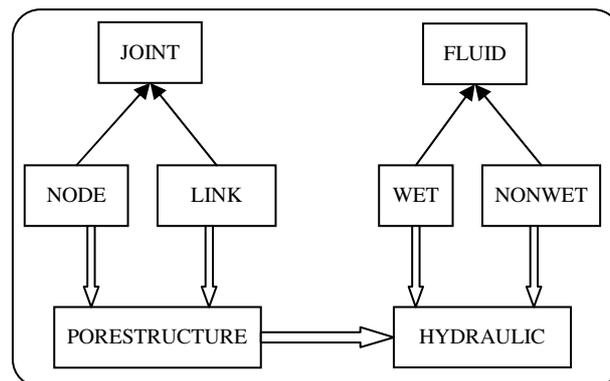


Fig. 1: Class hierarchy used in model programming.

methodology, a network is constructed by using object oriented design as schematically shown in Fig. 1. In this design, pores and throats are defined as 'NODE' and 'LINK' classes which are both derived from a 'JOINT' class. The 'JOINT' class contains all of the common information related to the pores and throats. These information include pore-throat radius, wettability (which is defined by fluid-solid contact angle), a list of functions which are used to calculate the area and volume of wetting and non-wetting phases for different geometrical shapes. In this design, each pore and throat knows all specific information related to itself. For a pore, these information include the list of all connected throats, pore geometry ($[x, y, z]$ in Cartesian coordinate system), pore length which is assumed to be twice as the pore radius. On the other hand, each throat knows the addresses of the two pores located at its both ends, therefore; the throat length can be calculated as follows:

$$L_t = \sqrt{((x_{p_2} - x_{p_1})^2 + (y_{p_2} - y_{p_1})^2 + (z_{p_2} - z_{p_1})^2)} \quad (1)$$

Where (x, y, z) denotes the coordination of the two pores located at both ends of the throat.

To assign volume and cross-sectional area to pores and throats, their geometry and shape must be specified. In this design, each 'JOINT' (pore/throat) may have cross sections with any geometrical shapes including circular, rectangular or triangular or n-sided polygons. In such cases, the cross-sectional areas are given by:

$$A_{\text{JOINT}} = C_{\text{geom}} r_{\text{JOINT}}^2 \quad (2)$$

Where C_{geom} is either π , 4, or $3\sqrt{3}$ depending on whether 'JOINT' cross-section is circular, rectangular or triangular respectively [27, 28]. The volume of each pore

and throat is simply calculated as follow:

$$V_{\text{JOINT}} = A_{\text{JOINT}} L_{\text{JOINT}} \quad (3)$$

Furthermore, a main class, which contains all the information about the structure of the network, is called 'PORESTRUCTURE'. This class contains all of the relevant network properties such as network dimension, a list of all pores, a list of all throats, a list of connected pores and the functions which are going to build the network from different statistical approaches (such as; Gaussian, Weibull, and Normal). Moreover, it includes the list of pores located at the inlet and outlet boundary of the network. The network can have a cylindrical or spherical shape. Therefore; the network bulk volume for both cases can be calculated as follow

For a rectangular network,

$$V_{\text{bulk}} = L_x \times L_y \times L_z \quad (4)$$

and for a cylindrical network,

$$V_{\text{bulk}} = \pi r_n^2 h, \quad (5)$$

Where L_x , L_y , L_z are the network length in x, y, z directions, r_n is the network radius and h is the network height. As the volume of each node and link, from Equations (2), (3), is known, by proper scaling of the pore and throat radii, a medium can be created with a specific porosity. Total porosity of the medium is calculated as follow:

$$\Phi_{\text{tot}} = \frac{V_{\text{pore}}}{V_{\text{bulk}}} \quad (6)$$

$$V_{\text{pore}} = \sum_{i=1}^{n_p} V_{p_i} + \sum_{i=1}^{n_t} V_{t_i} \quad (7)$$

where n_p , n_t and V_{pore} denote total number of pores and throats and pore volume respectively. In porous media, due to some diagenesis effect, the total porosity is higher than the effective porosity. In this design, since the pores and throats are randomly distributed inside the network, it is usual to find some dead end pores which are not connected to inlet and outlet boundary of the network and this seems to be similar to what happens in real samples. The volume of the dead end pores are not included in effective porosity calculations and represent the difference between the total and the effective pore volume. Therefore, effective porosity is defined as follows:

$$\Phi_{\text{eff}} = \frac{\sum_{i=1}^{n_{pc}} V_{p_i}^c + \sum_{i=1}^{n_{tc}} V_{t_i}^c}{V_{\text{bulk}}} \quad (8)$$

Where $V_{p_i}^c$ and $V_{t_i}^c$ denote the volume of connected pores and throats respectively. In the proposed framework, all of the information related to fluid such as, fluid type (wet, non-wet), fluid viscosity, surface tension and the properties of wetting and non-wetting phases are defined in 'FLUID' class. Two instances of 'FLUID' class and the required functions to calculate capillary pressure and relative permeability along with the methods drainage and imbibition processes are defined in 'Hydraulic' class. Furthermore, this class should have all the information related to the porous medium; therefore an instance of 'PORESTRUCTURE' class is allocated in this class. Since the total volume and the volume of occupying fluids (wet/non-wet) are known for each pore and throat, the saturation of both phases can be calculated:

$$S_w = \frac{\sum_{i=1}^{n_p} V_{p_{wi}} + \sum_{i=1}^{n_t} V_{t_{wi}}}{V_{\text{pore}}}, \quad S_{nw} = 1 - S_w \quad (9)$$

Where $V_{p_{wi}}$, $V_{t_{wi}}$ are volumes of wetting phase in pore and throat respectively

FLOW MECHANISM

For simulation of fluid flow in a network, invasion percolation theory is used. This method begins with assigning random number to each pore with an arbitrary distribution method [29, 30]. At the beginning, the network is filled with the defending fluid and the invading fluid will be injected from one face to the network. In the proposed model two incompressible fluids (oil/water) have been used; hence the defending fluid (oil or water) can be trapped by a portion of invading fluid in some parts of the network. Therefore, several factors affect the flow mechanism when invading fluid advances in the porous medium. In particular, wettability (the tendency of porous medium for keeping one fluid in comparison with another) is an important parameter and must be taken into account [31, 32].

The flow of the fluid in the porous media is governed by drainage/imbibition phenomena. The imbibition

is the phenomenon by which a wetting fluid advances into a porous medium spontaneously; whereas the displacement of wetting fluid by a non-wetting fluid is called drainage. These two phenomena are briefly described by many researchers [33-35].

The conventional method for measuring fluid saturation or searching trapped region is proposed by *Hoshen-Kopelman* [36]. In this method the entire network is searched and after labeling the connected parts, those pores that are connected to the outlet face of the network are known as potentially invaded pores. This approach is repeated in each time step even with a small change in the interface, which is time consuming and is considered as the main drawback of this method.

As previously described, each pore knows all the connected throat and neighboring pores, hence by starting from the inlet face of the network, the capillary pressure of all neighbors of each newly invaded pore or throat is checked. The invasion happens when the capillary pressure exceeds its threshold value. This process continues until the boundary is reached. Therefore, knowing all the invaded pores and throats, the wetting and non-wetting phase saturations can be calculated. Several sample wide searches were used to update the continuity of the fluid pattern and to label the new connected pores as necessary. This approach covers the entire possible local changes of the interface almost in all instances, since the invasion phenomenon happens once for each pore or throat.

OPTIMIZATION

For optimization of system connectivity, a non-gradient based optimization algorithm (random walk algorithm) is used [37]. After the pore network model is constructed, the match between the network results and experimental data is checked. If there is a difference, the network connectivity is adjusted in an ad-hoc manner until a good match is obtained. Various instances of the network are built with different series of random numbers and the best realization (the network with the minimum error) has been selected as the representative model of the target rock sample. The process flow chart is shown in Fig. 2.

SIMULATION OF FLUID DISPLACEMENT

Relative permeability calculation involves both primary drainage and imbibition phenomena. In this

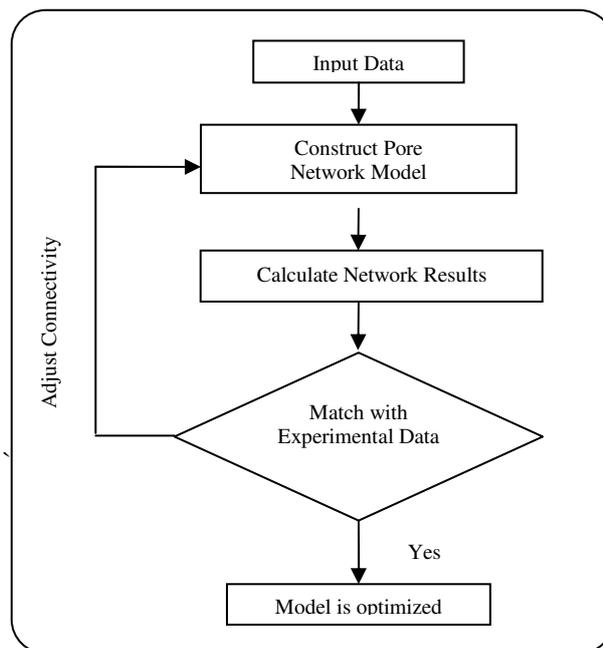


Fig. 2: The flow chart of optimization process.

study, a fluid flows at infinitesimal rates and capillary pressure controls the fluid pattern in the network. The pressure drop due to viscous forces assumed to be in the range at which Darcy law is applicable. To calculate relative permeability, the absolute permeability should be calculated; therefore Darcy's law is used to calculate the absolute permeability, k , of the model as follows [38]:

$$K = \frac{-\mu_p Q L}{A \Delta P} \quad (10)$$

where k is the absolute permeability, L is the distance and ΔP is the corresponding phase pressure drop between inlet and outlet boundaries of the network. A is the cross-sectional area perpendicular to flow direction and Q is the total flow rate of the fluid.

To calculate absolute permeability, first it is assumed that the network is completely filled with only one phase. In steady state flow of an incompressible fluid, mass conservation for each pore body is described as:

$$\sum_{j=1}^z q_{ij} = 0 \quad (11)$$

where q_{ij} is the volumetric flow rate between pores i and j and z is the coordination number. For single-phase flow, the flow rate is given as:

$$q_{ij} = C_{ij}(p_i - p_j) \quad (12)$$

whereas for two-phase flow:

$$q_{ij} = C_{ij}(p_i - p_j - p_{cij}) \quad (13)$$

where the conductance, C_{ij} , is calculated using Poiseuille's law for flow in a pipe: [39].

$$C_{ij} = \frac{\pi r^4}{8\mu l} \quad (14)$$

where, r and l are the radius and the length of the throat respectively.

A pore might be connected to several other pores by throats. The conductance of each phase, through an assembly of two pores connected to each other by a throat, is considered to be the harmonic mean of the conductance to the phase through the pores and through the connecting throat [40]:

$$\frac{L_{ij}}{C_{ij}} = \frac{L_{tij}}{C_{tij}} + \frac{1}{2} \left(\frac{L_i}{C_i} + \frac{L_j}{C_j} \right) \quad (15)$$

where L_{ij} is the distance between the centers of two connected pores, C_{ij} is the conductance of the assembly to each phase, L_{tij} is the throat length, C_{tij} is the throat conductance to each phase, L_i and L_j are the half length of the pores and C_i and C_j are conductance of the pores i and j , respectively.

Substituting Eq. (12), (14) and (15) into Eq. (11) yields a set of equations for the unknown pore pressures of each pore that is represented by:

$$p_1 \times \sum_{j=1}^z \frac{c_{1j}}{L_{1j}} - \sum_{j=1}^z \frac{c_{1j}}{L_{1j}} p_j = 0 \quad (16)$$

$$p_n \times \sum_{j=1}^z \frac{c_{nj}}{L_{nj}} - \sum_{j=1}^z \frac{c_{nj}}{L_{nj}} p_j = 0$$

The resulting system of linear equations for the pore pressures is solved using the Conjugate Gradient method [41]. Once the pressure distribution was obtained, the total flow rate, Q , through the pore network is calculated (i.e. summation of flow rate in the pores which are located at the inlet or outlet boundaries of the network); consequently, the absolute permeability is calculated by Eq.(4). Relative permeabilities are computed only when the network contains more than one phase. They are calculated only for phases that have at least one network-

spanning cluster which is continuous from inlet to outlet. For relative permeability, a mass balance equation is constructed for each phase and the pressure for each phase is solved using the same procedure described for the calculation of absolute permeability. Once the phase pressure is found, the flow rate and permeability of each phase are calculated. Knowing absolute permeability of each phase, the relative permeability for each phase is calculated by dividing the phase permeability by the total network permeability as follows:

$$K_{rp} = \frac{K_p}{K} \quad (17)$$

PRIMARY DRAINAGE PROCESS

Initially, the network is saturated with water. Oil gradually invades the pores as capillary pressure increases. Invasion continues until the connate water saturation is reached. The inlet of the network is connected to the injected fluid, the oil phase in this case. The outlet of the network is connected to the displaced fluid that is the water phase. The filling sequence depends on the capillary pressure of the network. This process starts from draining the largest throat (with smallest capillary pressure) connected to inlet of the network. The next element to be filled is either the next largest throat or the pore that is connected to the throat just filled. The filling process continues until the highest capillary pressure is reached (the smallest throats are filled). During each step, once the oil phase reaches to the outlet boundary of the network, the hydraulic conductances of both phases are calculated and the pore pressure equations are solved for both the wetting and the non-wetting phases. In this model, it has been assumed that the wetting phase is continuous and flows between the non-wetting phase and the pore inner walls. The non-wetting phase conductance is calculated assuming it flows through the center of the pore. The radius of curvature between the two phases is obtained from Laplace's equation:

$$r_{ow} = \frac{\gamma_{ow}}{P_c} \quad (18)$$

Where γ_{ow} is the oil-water interfacial tension and P_c is the capillary pressure. The cross sectional area for each phase is calculated using the following equations obtained from the geometry of the pore [42]:

$$A_w = n_{\text{corners}} r_{\text{ow}} \left[\cot(\alpha) \cos^2(\alpha + \theta_{\text{ow}}) - \left(\frac{\pi}{2} - (\alpha + \theta_{\text{ow}}) \right) + \cos(\alpha + \theta_{\text{ow}}) \sin(\alpha + \theta_{\text{ow}}) \right] \quad (19)$$

$$A_{\text{nw}} = A_{\text{tot}} - A_w \quad (20)$$

Where n_{coenwes} is the number of corners, A_{nw} , A_w and A_{total} are cross-sectional areas occupied by non-wetting, wetting, and total phases respectively. α is a corner half angle of throat and θ is a fluid-solid contact angle. The conductance and effective radius for the bulk phase are given as follows:

$$C = \frac{A_w r_{\text{eff}}^2}{\beta \mu L_t} \quad \text{and} \quad (21)$$

$$r_{\text{eff}} = \sqrt{\frac{A_{\text{nw}}}{\pi}} \quad (22)$$

Where β is a dimensionless resistance factor for the flow of wetting liquid in pore corners [43]. The total conductance for the non-wetting phase between pores i and j is calculated using Eq. (21). The total conductance involves both the bulk flow through the center of water filled pores with C_{ij} explained above and the corner flow in pores. Then the mass balance equation for each pore is solved and the pore pressure for each phase is calculated as discussed above. Subsequently, the total flow rate and the relative permeability for each phase can be calculated.

IMBIBITION PROCESS

The procedure used for imbibition is similar to what used in primary drainage, water is allowed to imbibe into the oil filled pores. Water is connected to the inlet of the system and the capillary pressure decreases incrementally. Initially capillary pressures for all the possible filling elements should be calculated and ranked in the decreasing order. The one with the smallest capillary pressure is the last one to be filled with water. After the elements (pores or throats) are filled, the front of wetting phase is changed in the whole system and the same calculations should be repeated. Therefore, capillary pressures for the recent and the last filled elements are ranked again. This process continues until no element can be filled any more. Another point which should be carefully taken into account is the location of the trapped oil which should not be changed in the network. Therefore, the oil path is monitored to make

sure that the oil is moveable. Water is a continuous phase in the system and can always flow. Imbibition displacement is more complicated and involves two main mechanisms including snap-off and piston-type displacement. [44, 45].

Piston-like displacement

This is the process by which the invading fluid pressure is high enough to allow it to enter the bulk of the element (pore or throat) by pushing the displaced fluid in front of it. This mechanism is the default and assumed to be dominant until the criterion for snap-off is met.

Snap-off mechanism

Snap-off corresponds to an imbibition event where the non-wetting phase in the centre of a pore or throat is displaced by the wetting phase residing in the corners or layers. This occurs when the capillary pressure decreases or the radius of the curvature of the fluid interface increases and the wetting layers in the crevices start to swell. The swelling of the wetting layers continues to a point where further filling of the crevices causes the interfacial curvature to decrease which leads to fluid instability and the centre of the pore space is spontaneously filled with wetting fluid [46]. The importance of pore geometry by considering pores with angular cross-section and its effect on snap-off was first described by *Pickell et al.* [47]. Subsequently, *Roof* [48] showed that snap-off occurs when the interfacial configuration cannot satisfy the pressure distribution in the phases. Snap-off is controlled by aspect ratio (the ratio between the pore radius to throat radius). In this study snap-off happens when the aspect ratio is larger than 3 which is a criterion proposed by Roof.

EXPERIMENTAL STUDY

The objective was to obtain the saturation profile, absolute and relative permeability and capillary pressure in order to determine fluid displacement mechanisms during co-current fluid flow into oil (water)-filled cores. X-ray CT scanning was used for visualization of the displacement patterns and validation of the model. Saturation variations along the plug length were measured by using the X-ray absorption method. The porous medium is a well characterized carbonate

reservoir core sample from field "AZN" in south west of Iran. Tests were performed at Research Institute of Petroleum Industry (RIPI) laboratories of National Iranian Oil Company (NIOC). The experiments were performed at a constant temperature of (25 °C) and a pressure close to the atmospheric pressure. The sample used in this work, had a cylindrical shape with a circular cross section of 1.5 in, (38 mm) and a length of 3 in. (76 mm) and was laterally coated. Average sample porosity measured from X-ray absorption profiles was 0.18 [49]. Afterwards the sample was dried and saturated with brine; the porosity and the absolute permeability to brine were measured as 0.18 and 30 md respectively. In the next step, oil was injected into the sample and in order to obtain a more uniform saturation profile the direction of flow was changed alternatively several times. Finally capillary pressure and relative permeability data for both drainage and imbibition processes were gathered by performing the relevant tests (centrifuge method for P_c and steady state method for K_r) at laboratory condition.

RESULTS AND DISCUSSION

To validate the ability of the proposed framework to model porous media, the proposed model has been used to construct the network model for a carbonate rock sample. The three-dimensional pore network built here is shown in Fig. 3 This network represents a 1.5 mm × 1.5 mm × 1.5 mm sample of carbonate field "AZN" in southwest of Iran. It consists of 797 pores (nodes) and 2235 throats (links). A larger pore network size is certainly possible, but uses much more CPU time. In this model, random distribution for contact angles between (0-35°) is imposed. Based on actual data obtained from Scanning Electron Microscopy (SEM) pictures and image analysis, most of the pores (75 %) are triangular in cross-section, near 15 % of the pore bodies have circular cross-sections and 10 % are rectangular. For throat, most of the throats are triangular in cross section and just twenty percent of them have rectangular and circular cross sections. The numbers of pores connected to the inlet and to the outlet are 78 and 82 respectively. The network total porosity is 0.1787 and the absolute permeability is 30 md.

Input to the model

To construct the pore network model for the mentioned

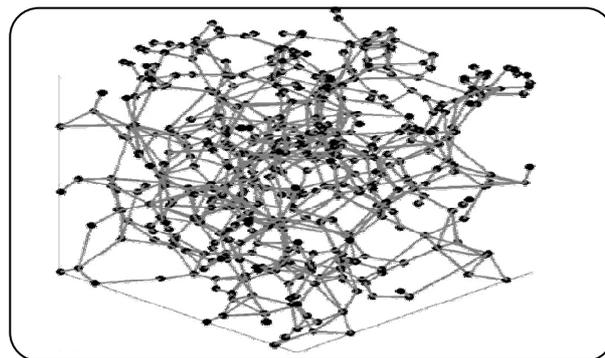


Fig. 3: A picture of 3D network used in this work.

sample, experimentally measured capillary pressure data (using mercury injection method) was used to obtain the pore size and throat size distribution.

In this model, throats radii were assigned according to the truncated Weibull distribution [26, 50]:

$$r_t = (r_{t,max} - r_{t,min}) \left[-\delta \ln \left(x \left(1 - e^{-1/\delta} \right) + e^{-1/\delta} \right) \right]^{1/\gamma} + r_{t,min} \quad (23)$$

Where R_t is the radius of a throat, x is a random number between zero and one, and δ and γ are fixed parameters determined by experimental measurement.

The pore radius is defined as follows:

$$r_p = \max\{r_{i_i} | i=1, \dots, n\} \times a \quad (24)$$

where n is the number of the connecting throats and the aspect ratio a is the ratio between the pore radius and the maximum radius of the connecting throats. This parameter is also distributed according to truncated Weibull distribution as follows:

$$a = (a_{max} - a_{min}) \left[-\delta \ln \left(x \left(1 - e^{-1/\delta} \right) + e^{-1/\delta} \right) \right]^{1/\gamma} + a_{min} \quad (25)$$

For considering different pore (throat) shapes in the model, the shape factor ($G = \text{area} / (\text{perimeter}^2)$) and the contact angle (θ) for each pore and throat are calculated based on the method discussed previously (truncated Weibull distribution). In this study, Water (with the density of 1.0 g/cc and viscosity of 1.14 cp) and oil (Kerosene with the density of 0.79 g/cc and viscosity of 1.22 cp) are used as a wetting and non-wetting phases respectively. The interfacial tension between wetting and non-wetting phases is 30 mN/m.

FRAMEWORK ACCURACY ANALYSIS

The results for capillary pressure and relative permeability computations in both drainage and imbibition processes are obtained based on the pore network model constructed using the experimentally measured pore and throat data. The effective radii and the lengths of the pores and throats through the model are distributed statistically, using the mercury injection data shown in Fig. 4. As this figure shows, pore- throat size distribution in this rock is bimodal (note, the possibility more than 18% is considered to be one mode in this study) with local maximums at roughly 0.08 and 2 μm . As previously described, the shape factor and other input data in model are distributed according to the truncated Weibull distribution method as shown in table 1.

In this work, relative permeability and capillary pressure are obtained by simulation of both primary drainage and imbibition displacement for two-phase flow in a water-wet system. The assumptions made are that the fluids flow at an infinitesimally rate, viscous pressure drop is negligible, and capillary pressure controls fluid invasion and imbibition. Therefore, a quasi-static model is used [51, 52]. Then the connectivity of the whole system has been optimized, by using a nongradient based optimization algorithm. This technique works well for both primary drainage, and the imbibition displacement. After the pore network model of the sample is constructed, the match between the computed drainage capillary pressure curve and the mercury injection measurement has been checked and the network connectivity is adjusted iteratively until a good match is obtained. After the capillary pressure curve is matched, the relative permeability curves for both drainage and imbibition displacements have been computed and compared with experimental data. The obtained results from the network model are shown in Figs. 5 -11. As Fig. 5 shows, for water saturation more than fifty and less than twenty percents the results of drainage capillary pressure data from the model matches the experimentally measured drainage data with good agreement. Moreover, Fig. 5 shows the results of drainage capillary pressure against experimental measurements. It can be seen in this figure that the obtained results are in good agreement with experimental data. In imbibition, as Fig. 6 shows, the model output predicted experimentally measured data with good agreement in whole range of

Table 1: Weibull Distribution Parameters Used in Model.

Parameters	Min.	Max.	γ	δ
Throat Radius(R_t), μm	0.01	15	0.8	0.5
Aspect Ratio(a)	3.0	0.7	3.0	0.2
Contact angel (θ), $^\circ$	0	35	2.0	0.3
Shape Factor(G)	0.07958	0.001	1.6	0.8

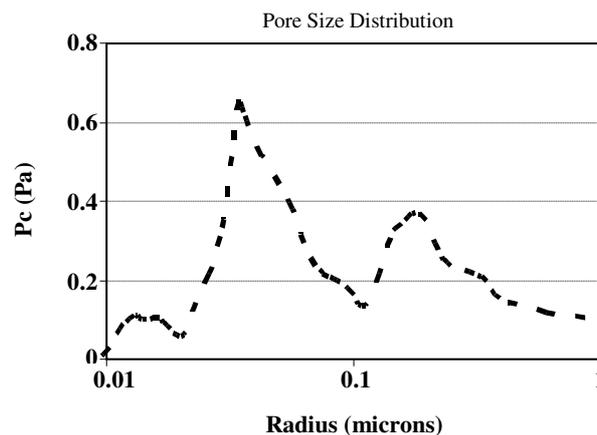


Fig. 4: Pore-Throat size distribution from mercury injection test.

water saturation. This may happen due to considering snap-off mechanism in imbibition process.

In order to verify the relative permeability from the model, it has been compared with core-scale drainage and imbibition experimental data; as shown in Figs. 7 and 8. As Fig. 7 shows, the model prediction of drainage relative permeability of wetting phase has a good agreement with measured data while the result for non-wetting phase show a little difference that is not significant. It is obvious that producing imbibition relative permeability data in imbibition process is more difficult than drainage process. These difficulties become more significant when the core plugs are water wet and low permeable and therefore a few K_r data have been measured for selected plug. However, these data are predicted by the model successfully as Figs. 8 shows. Furthermore, it should be mentioned that these reasonable matches for relative permeability data in both drainage and imbibition processes are obtained with no parameter adjustment.

The pore network model results for the saturation variation along the plug are also compared with the core-

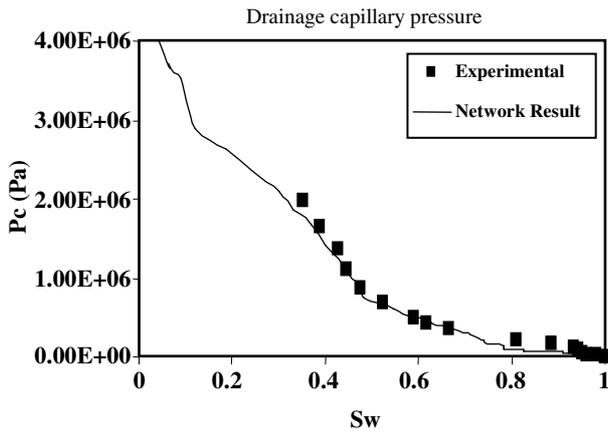


Fig. 5: Comparison of the network capillary pressure to experimental data during primary drainage.

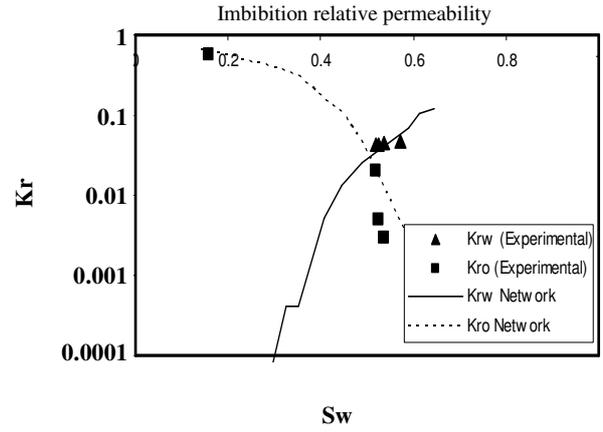


Fig. 8: Comparison of the network relative permeability to experimental data during primary imbibition.

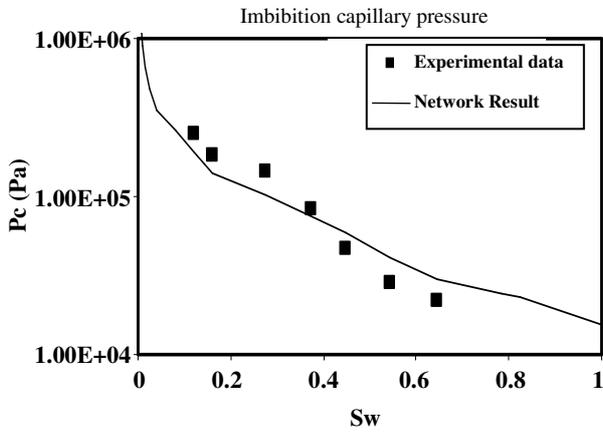


Fig. 6: Comparison of the network capillary pressure to experimental data during primary imbibition.

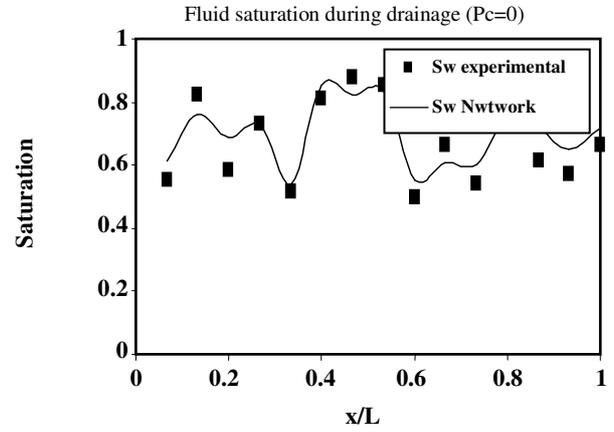


Fig. 9: Comparison of the network residual Water saturation variation along the plug with experimental data during drainage process at $P_c = 0$.

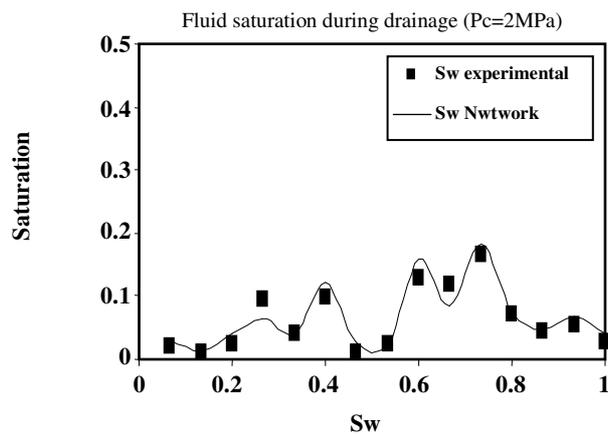


Fig. 7: Comparison of the network relative permeability to experimental data during primary drainage.

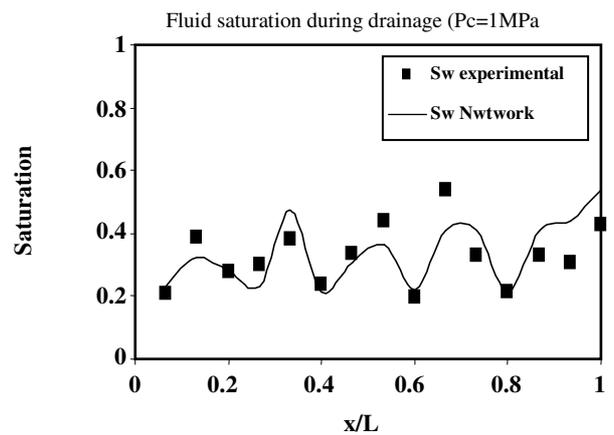


Fig. 10: Comparison of the network residual water saturation variation along the plug with experimental data during drainage process at $P_c = 1$ MPa.

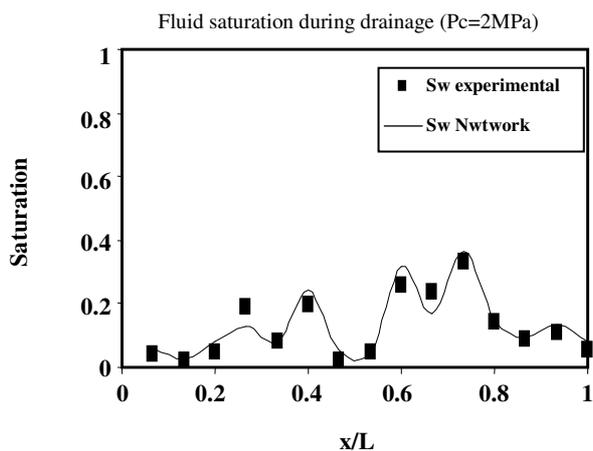


Fig. 11: Comparison of the network residual water saturation variation along the plug with experimental data during drainage process at $P_c = 2$ MPa.

scale experimental data for both drainage and imbibition processes. The results are shown in Figs. 9, 10 and 11. As these figures show, there is a reasonable agreement between the network results and the experimental data. As Fig. 10 shows the quality of matches is going to be better at higher capillary pressure values and this can be a good indication to show the ability of the model to predict residual fluids saturation. A low permeable carbonate plug as a complicated porous media sample has been selected in this case study. The result of simulation show good prediction of the model in comparison with experimental data. These results proved that, this framework has enough ability to model the complicated porous medium and it is quite powerful to extrapolate its microscopic properties by using the limited core-scale data.

It should be noted that, the criteria based on which the pore network model has been obtained is not a spatial criteria and the model is adjusted based on mismatched that exist in relative permeability and capillary pressure data. No one can claim that the obtained model is the unique representative of the sample. Since the obtained model fulfills all available experimental data, it is the best model that can be used.

CONCLUSIONS

In this paper we have mainly focused on the way in which the constructed network can reasonably represent the geometry and topology of the pore space. To build such a network model an object oriented framework

which inherently mimics phenomena taking place in the porous media has been proposed. The network model built on the basis of this framework has some advantages in comparison to regular lattice networks. This framework gives an opportunity to refine or improve some parts of the network without doing major change in the whole system. This will happen by adding or removing some parameters from different classes on the system. This ability increases the flexibility of the network to simulate the irregular porous medium and it can also be extended to cover several rock types with a variety of heterogeneities along the core plugs. In another word, this flexibility of the framework made it to be independent of lithology; therefore each type of lithology with an appropriate data can be modeled in this framework.

The major advantage of this methodology is that the constructed network doesn't have any restriction for the coordination number (number of connected throats to each pore). Hence, this number is not limited to a specific range and the network connectivity can be optimized by using an appropriate optimization algorithm (nongradient based optimization algorithm used in this work).

In this framework, it is possible to model any long range connectivity of pore space imposed by geological processes. Image analysis of SEM and experimentally available pore-throat distribution, relative permeability and capillary pressure data of a carbonate plug was used to verify the network model and the simulated relative permeability and capillary pressure results of the model. The results showed good agreement between the networks output and the experimental data. Therefore it was proved that this methodology is quite able to model complicated porous media by using limited experimental data.

Nomenclatures

A	Cross-sectional area
V	Volume
S	Saturation
g	Gravity acceleration
C	Hydraulic conductance
C_{geom}	Geometry factor
G	Shape factor
K	Permeability of the network
K_p	Absolute permeability of phase p

k_r	Relative permeability
L	Length
L_x, L_y, L_z	Network length in x-, y-, and z-direction
R	Radius
n_{corner}	Number of corners for given geometry
P	Pressure
P_c	Capillary pressure
Q	Volumetric flow rate
A	Aspect ratio
B	Resistance factor
γ_{ow}	Interfacial tension
Δ	Difference
θ	Contact angle
μ	Viscosity
Π	Constant, 3.141592654
ϕ	Porosity
x	Random number
b	Throat radii weight
α	A half corner angle of triangular throat
γ and δ	Weibull distribution function parameters
r_{ow}	Radius of curvature
Z	Coordination number

Subscripts

Abs	Absolute
eff	Effective
tot	Total
i,j	Property related to pore i/j
min	Minimum
max	Maximum
w	Wetting or water
nw	Non-wetting
o	Oil
t	Throat
p	Pore
ow	Oil-water
in	Inlet
out	Outlet
JOINT	Pore or throat

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